

Problem Set 1

Note: “CB” below means our reference book *Statistical Inference* written by Casella and Berger.

1. CB 1.6
2. CB 1.13
3. CB 1.26
4. CB 1.41
5. CB 1.51
6. CB 1.53
7. Suppose we have three missiles: missile 1, missile 2 and missile 3. Suppose each missile is able to hit the enemy plane with probability of 0.7.
 - (i). Without extra information, give a lower bound for the probability of the event “all missiles hit the enemy plane”.
 - (ii). If the 3 missiles are launched independently, what’s the probability that the plane is hit?
8. Suppose X is a random variable taking integer values, $\Pr(X \geq 2) = a$ and $\Pr(X \leq 2) = b$. Find $\Pr(X = 2)$ in terms of a and b .
9. Consider the random experiment of rolling a dice, which gives rise to the sample space

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let

$$\mathcal{B} = \{S, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{3, 4, 5, 6\}\}.$$

Is $\mathcal{B}$ a sigma-algebra? Explain why.

10. Suppose some patient walks into a hospital to get tested for a disease, which is affecting 1% of the total population. Let D denote the event: this patient has the disease. Let T denote the event: this patient’s test is positive. Suppose the nurse uses a testing procedure that claims to have 95% accuracy, i.e.,

$$\Pr(T \mid D) = \Pr(T^c \mid D^c) = 0.95.$$

- (i). Find $\Pr(D \mid T)$.
- (ii). Suppose the proportion in the total population affected by the disease is γ (instead of 1%). Find $\Pr(D \mid T)$ as a function of γ . How does it change with γ ?